Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Mathematics-I
Subject Code : 4SC01MAT1 **Branch : B.Sc. (All)**
Semester : 1 **Date : 16/03/2019** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) What is order of differential equation : $(y'')^5 + y''' + 2xy = 0$ (1)

b) The solution of differential equation $y'' + 2y = 0$ is ……. (1)
(a) sinx (b) cos2x (c) 2sinx (d) 2cosx

c) True/false : Machlaurin's series is particular case of taylor's series. (1)

d) Write the equation of circle having centre (0 ,0) and radius 5 in polar form. (1)

e) Find 5^{th} derivative of log(2x+7). (1)

f) True/false: Every system of linear simultaneous homogenous equation is consistent. (1)

g) Define: Order and degree of differential equation . (2)

h) Explain the difference between order and rank of matrix. (2)

i) Give an example of differential equation which is exact also justify it. (2)

j) Find order and degree of the differential equation (2)

$$\left(\frac{d^8y}{dx^8}\right)^5 + \left(\frac{d^7y}{dx^7}\right)^2 + 2xy = 0.$$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find rank of matrix: **(5)**

$$\begin{bmatrix} 3 & 4 & -2 & 1 & 2 \\ 7 & -3 & 1 & -2 & -1 \\ 3 & 1 & 0 & 1 & 3 \end{bmatrix}.$$

b) Solve $7x - 3y + 2z = 11, 4x - 6y - 2z = 15, 7x + 3y - 4z = 1$ using Cremer's method. **(5)**

c) Find Eigen value of **(4)**

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 2 & 7 \\ 7 & 7 & 1 \end{bmatrix}.$$

Q-3 **Attempt all questions** **(14)**

a) Discuss the consistency of the system of equation **(5)**

$2x - 3y + 5z = 11, 3x - 4y + 7z = -12, 4x + 10y + 17\,z = 5.$

If it is consistent then find it's solution.

b) Find characteristic equation of matrix **(5)**

$$\begin{bmatrix} 3 & 2 & 3 \\ 0 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

Using it find value of $A^8 - 6A^7 + 5A^6 - 3A^5 + 5A^4 - 8A^3 - 2A + I.$

c) If $A = \begin{bmatrix} 30 & 2 \\ 10 & 4 \end{bmatrix}$ then verify Caley Hamilton's theorem. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Solve: $(x^2 + y^2)dx - 2y^2\,dy = 0.$ **(5)**

b) Solve: $\frac{dx}{dy} + \frac{4y}{y^2+1}\,x = \frac{1}{(y^2+1)^3}$ **(5)**

c) Solve: $x\cos y\,dx - \frac{x^2}{2}\sin y\,dy = 0$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Find equation of sphere which passes through $(0,0,0), (3,0,0), (0,4,0)$ and $(0,0,6)$. **(6)**

b) State and prove Leibnitz's theorem for n^{th} derivative of product and find nth derivative of $x\sin x$. **(8)**

Q-6 **Attempt all questions** **(14)**

a) Find n^{th} derivative of the following : **(6)**

(a) $\frac{1}{(2x+1)(2x+2)}$ (b) $\frac{2x+1}{x^2-1}$

b) State n^{th} derivative of $\sin ax$ and a^x and prove it. **(4)**

c) If $y = e^{ax}\cos(bx+c)$ then show that $y_n = (a^2 + b^2)^{\frac{n}{2}}\, e^{ax}\cos(bx+c+n\tan^{-1}\left(\frac{b}{a}\right))$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Express $x^5 + 4x^4 + 6x^3 - 4x + 1$ as powers of $x - 2$. **(5)**

b) Find machlurin's series of $\sinh x + \cosh x$. **(5)**

c) Express $\sin x \cdot \cos x$ in powers of x upto x^6 . **(4)**

Q-8 **Attempt all questions** **(14)**

a) Evaluate the following : **(6)**

(1) $x \xrightarrow{lim} \infty \ \ (e^{x+e^{-x}} - e^{x})$.

(2) $x \xrightarrow{lim} \frac{\pi}{2} \ \ (sinx)^{tanx}$.

b) Apply Rolle's theorem for f(x)= (2x-1)sinπx in the interval [$\frac{1}{2}$, 1]. **(6)**

c) Define: Taylor's series . **(2)**